Semantics and the Plural Conception of Reality*

Salvatore Florio

Kansas State University and Birkbeck, University of London
florio@ksu.edu

Abstract

According to the singular conception of reality, there are objects and there are singular properties, that is, properties that are instantiated by objects separately. It has been argued that semantic considerations about plurals force us to embrace a plural conception of reality. On the plural conception of reality, there are plural properties — properties that are instantiated by many objects together — alongside singular ones. I propose and defend a novel semantic account of plurals which dispenses with plural properties and, thus, undermines the semantic argument in favor of the plural conception of reality.

1 Introduction

According to a traditional view, reality is singular. Socrates and Plato are philosophers. Each of them has the property of being a philosopher.¹ Being a philosopher is a singular property in that it is instantiated separately by Socrates and by Plato. The property of being a philosopher, like the property of being human, has the higher-order property of being instantiated. But the property of being instantiated is singular too. It is instantiated separately by the property of being a philosopher and by the property of being human. If we generalize these ideas, we obtain what may

¹For helpful comments and discussion, I wish to thank Andrea Borghini, Ben Caplan, Simon Hewitt, Øystein Linnebo, Tom McKay, David Nicolas, Alex Oliver, Agustín Rayo, Marcus Rossberg, Richard Samuels, David Sanson, Stewart Shapiro, Timothy Smiley, Florian Steinberger, William Taschek, Neil Tennant, Gabriel Uzquiano, and Sean Walsh. Earlier versions of this article were presented at The Ohio State University, the Sociedad Argentina de Análisis Filosófico, the Université de Genève, the Central Division of the American Philosophical Association, and Birkbeck College. I am grateful to the members of the audiences for their valuable feedback.

¹Those who prefer to understand predication in terms of Fregean concepts should have no difficulty in translating claims about properties into claims about concepts throughout the article.
be called the singular conception of reality. On this conception, reality encompasses entities belonging to two main categories: objects and singular properties of different orders.\(^2\)

The singular conception of reality offers a simple picture of what there is and it also offers a simple picture of the semantics of singular predication. A basic predication of the form \(S(t)\), composed of a singular term \(t\) and a singular predicate \(S\), is true in a given interpretation of the language if and only if, relative to that interpretation, the object denoted by \(t\) instantiates the property denoted by \(S\).

A broader conception of reality, however, has been advocated. The Romans conquered Gaul. This not something that any Roman did separately. They conquered Gaul together. According to advocates of the broader conception of reality, conquering Gaul is a plural property, one that is instantiated together by the Romans. We may call the broader conception of reality, which admits the existence of plural properties alongside singular ones, the plural conception of reality.\(^3\)

An attractive feature of the plural conception of reality is that it offers a straightforward way to handle plural predication. Consider an atomic formula \(P(tt)\) composed of a plural term \(tt\) and a plural predicate \(P\). If plural properties are available, one may hold that \(P(tt)\) is true in a given interpretation of the language if and only if, relative to that interpretation, the things denoted by \(tt\) instantiate together the plural property denoted by \(P\).

It is far from clear how one should account for plural predication in the absence of plural properties. Does the singular conception have the resources to provide a satisfactory semantics for plurals? Or do semantic considerations force us to abandon it in favor of the plural conception? This is the semantic challenge to the singular conception.

It has been maintained that the singular conception cannot meet the semantic challenge and various authors have embraced the plural conception on semantic grounds.\(^4\) I think that we have reasons to resist this view. In this article, I argue that the semantic challenge to the singular conception can in fact be met. I propose and defend a novel approach to the semantics of plurals which dispenses with plural properties and, thus, undermines the semantic argument in favor of the plural conception. On this approach, plural terms denote singular properties and plural predicates denote singular higher-order properties. While articulating my proposal, I take a fresh look at the semantic debate about plurals: I clarify what is at stake

---

\(^2\)I use ‘entity’ as an umbrella term covering both objects and properties, and I take ‘object’ to mean the same as ‘individual’.

\(^3\)The label was suggested by Yi (2006).

between competing positions and, in light of distinctions that have been overlooked, I give a more complete characterization of the territory.

2 Objections to singularism

A simple but problematic answer to the semantic challenge to the singular conception relies on the following view.

\[ \text{REGIMENTATION SINGULARISM} \]

Plural terms and predicates are not required in the regimentation of natural language into a formal language. Singular terms and predicates suffice for the task.

\[ \text{REGIMENTATION SINGULARISM} \] is a clear ally of the singular conception. The regimentation singularist contends that plural constructions can be rendered by paraphrase in a regimenting language containing only singular expressions, and there is no obstacle in giving a semantics for such a language within the singular conception. The task facing the regimentation singularist is to provide a good method of paraphrase.

Two methods suggest themselves. According to the first, plural expressions are paraphrased in terms of set-theoretic expressions. According to the second, plural expressions are paraphrased in terms of mereological expressions.\(^5\) However, there are familiar objections to both methods. The main objection is that they yield implausible entailment relations. For instance, on both approaches the truth of a sentence such as (1) would logically entail (2).

(1) Russell and Whitehead cooperate.

(2) There is a set or there is a mereological sum.

But (2) does not appear to be a logical consequence of (1).

A number of additional arguments have been marshaled against these construals of \text{REGIMENTATION SINGULARISM}. This sentence expresses a set-theoretic truth:

(3) There are some sets such that any set is one of them if and only if it is not self-membered.

The set-theoretic version of \text{REGIMENTATION SINGULARISM} seems bound to paraphrase (3) as (4).

\(^5\)For classic proposals of the mereological approach, see Massey (1976) and Link (1983).
(4) There is a set \( x \) such that, for any set \( y \), \( y \) is in \( x \) if and only if \( y \) is not self-membered.

However, (4) is inconsistent with very modest set-theoretic principles. Thus, a set-theoretic truth such as (3) is rendered equivalent to a set-theoretic statement that is generally taken to be false.\(^6\)

Moreover, the set-theoretic version of REGIMENTATION SINGULARISM falls prey to a general Russellian argument, which has been called the “paradox of plurality”\(^7\). According to the axiom schema of plural comprehension, for any singular predicate \( S \):

(5) If something \( S \), then there are some things such that anything is one of them if and only if it \( S \).

Suppose that one chooses the singular relation \( K \) to paraphrase the predicate ‘being one of’ occurring in (5) and suppose that there is another predicate \( R \) in the object language that is also paraphrased as \( K \). Finally, suppose that \( R \) is not reflexive in the sense that (6) holds.

(6) Something does not \( R \) itself.

Then (6) and plural comprehension applied to the predicate ‘does not \( R \) itself’ entail:

(7) There are some things such that anything is one of them if and only if it does not \( R \) itself.

The set-theoretic paraphrase of (7) is (8).

(8) There is a set \( x \) such that for every \( y \), \( y \) \( K \) \( x \) if and only if \( y \) does not \( K \) \( y \).

But (8) is inconsistent. Any non-reflexive paraphrase of ‘being one of’ becomes problematic as soon as the same paraphrase is given to another singular predicate in the object language. In particular, the argument goes through when both \( K \) and \( R \) are replaced by the relation of set-theoretic membership.\(^8\)

The mereological construal of REGIMENTATION SINGULARISM escapes the paradox of plurality. In the mereological framework ‘being one of’ is typically rendered as parthood, which is reflexive. So the paradox does not arise. However, other arguments have been leveled against this view. For the mereological singularist, the

---


\(^7\)Higginbotham (1998), p. 265.

\(^8\)Discussions of the paradox of plurality are found, for example, in Schein (1993), Chapter 2, and Oliver and Smiley (2001).
natural rendering of a plural term \(tt\) is ‘the sum of \(tt\)’. For instance, ‘Russell and Whitehead’ is paraphrased as ‘the sum of Russell and Whitehead’.\(^9\) Now consider this identity claim:

\[\text{(9) The sum of Russell and Whithead is identical to the sum of the molecules of Russell and Whitehead.}\]

As suggested by Oliver and Smiley (2001), by accepting (9), the mereological singularist would be committed to (11) on the basis of (10).\(^10\)

\[\text{(10) Russell and Whitehead were logicians.}\]

\[\text{(11) The molecules of Russell and Whitehead were logicians.}\]

A similar argument has been put forth by Rayo (2002). Imagine that we have some sand and that the grains of sand are grouped into piles. There is a possible scenario in which both of the following sentences are true.

\[\text{(12) The piles of sand are scattered.}\]

\[\text{(13) The grains of sand are not scattered.}\]^11

Now, the piles of sand form the same mereological sum as the grains of sand. So (14) holds.

\[\text{(14) The sum of the piles of sand is the sum of the grains of sand.}\]

But, given (14), the mereological singularist must implausibly regard (12) and (13) as contradictory.\(^12\)

Schein (1993) has pointed out that the mereological version of REGIMENTATION SINGULARISM gets the mereological facts wrong. Define an atom to be an individual that does not have proper parts. The expression ‘the atoms’ is paraphrased as ‘the sum of the atoms’. Likewise, ‘the sums of atoms’ is paraphrased as ‘the sum of the sums of atoms’. But, whatever the domain, the sum of the atoms is identical to the sum of the sums of atoms, as they are both the sum of everything in the domain. So, from (15), one is forced to conclude (16).

\[^9\text{Since plurals must be completely eliminated in the paraphrase, ‘the sum of Russell and Whitehead’, which is only partially singular, should be taken to stand for a fully singular expression defining the relevant notion of sum, e.g., ‘the object } x \text{ such that anything overlaps } x \text{ if and only if it overlaps Russell or it overlaps Whitehead’. For the sake of exposition, the arguments in the remainder of this section are formulated using partially singular paraphrases.}\]

\[^10\text{Oliver and Smiley (2001), p. 293.}\]

\[^11\text{The scenario envisioned is one in which the grains of sand are nicely grouped into piles — hence they are not scattered — while the piles themselves are scattered.}\]

\[^12\text{Rayo (2002), pp. 444-445.}\]
(15) The atoms are exactly two.

(16) The sums of atoms are exactly two.

We know that if there are \(n\) atoms, the number of mereological sums over them is \(2^n - 1\). Therefore, if the number of atoms is two, there are three sums of atoms, which is inconsistent with (16).

The arguments just reviewed concern two versions of REGIMENTATION SINGULARISM. Regimenting plural sentences within a singular language containing set-theoretic or mereological vocabulary is, as we have seen, problematic. As a result, this view has now become prominent.

REGIMENTATION PLURALISM
Plural terms and predicates are required in the regimentation of natural language into a formal language.

It is important to notice, however, that appealing to REGIMENTATION SINGULARISM is only one way for the proponent of the singular conception to address the semantic challenge. The singular conception does not fall with REGIMENTATION SINGULARISM. REGIMENTATION PLURALISM is indeed compatible with what may be called SEMANTIC SINGULARISM.

SEMANTIC SINGULARISM
Semantic interpretations for plural sentences can be specified adequately within a singular framework.

The focus has now shifted from the level of regimentation to the level of semantics. The semantic singularist need not paraphrase plural constructions into singular ones. She can employ plural expressions in the regimenting language and take plural predication at face value instead of regarding it as singular predication in disguise. At the semantic level, though, she will draw exclusively on singular resources, characterizing the interpretations of the regimenting language without stepping outside the boundaries of the singular conception. Therefore, a successful response to the semantic challenge is available so long as there is a tenable version of SEMANTIC SINGULARISM.

Those who advocate the plural conception of reality on semantic grounds reject SEMANTIC SINGULARISM in favor of SEMANTIC PLURALISM.

SEMANTIC PLURALISM
Specifying semantic interpretations for plural sentences requires a plural framework.

\(^{13}\)For an extensive discussion of the prospects of the mereological approach, see Nicolas (unpublished). For a large-scale defense of it, see Link (1998).
Before turning to the dispute between semantic singularists and semantic pluralists, I will briefly introduce a regimenting language for plurals in the spirit of REGIMENTATION PLURALISM. This will be the object language of the competing semantic accounts that will occupy us from section 4 onwards. By adopting it, we are leaving REGIMENTATION SINGULARISM and its problems behind.

3 Regimenting plurals

Following the predominant practice in the philosophical literature, the regimenting language will be construed as an extension of first-order logic, including symbols for plural predicates, such as ‘cooperate’ or ‘are infinite’, plural variables and plural quantifiers, and a distinguished relational symbol for ‘being one of’, which will be treated as logical. This will be a version of the language known as PFO+.

Every plural predicate takes a fixed number of arguments, each of which can be exclusively singular or exclusively plural. An argument place is singular if it is occupied by singular terms, i.e., singular constants and singular variables. It is plural if it is occupied by plural variables. A predicate is said to be plural if at least one of its argument places is plural.

More specifically, the vocabulary of our language is composed by the usual vocabulary of first-order logic plus the following symbols.

A. Plural variables: \( vv \) and \( vv_i \) for each \( i \in \omega \).

B. Plural existential and universal quantifier (\( \exists, \forall \)) binding plural variables: \( \exists vv, \forall vv, \exists vv_1, \ldots \).

C. Symbols for collective plural predicates of any finite arity, with or without numerical subscripts: \( A, B, C, \ldots, A_1, A_2, \ldots \). For any arity \( n \), it is convenient to allow only predicate symbols whose first \( m \) argument places (\( 1 \leq m \leq n \)) are plural and whose remaining \( n - m \) argument places are singular. A superscript enclosed in square brackets will indicate the number of plural arguments. The arity of the predicate, when marked, will precede the square brackets. For example, if \( W^{2[1]} \) represents the two-place predicate ‘wrote together’, the sentence ‘they wrote Principia Mathematica together’ may be represented as \( W^{2[1]}(vv, p) \). For the sake of readability, I will often depart from this convention and allow the order of the terms to reflect the order found in English.

---


15 The double variables \( xx, yy, zz \) will be used as plural variables in the metalanguage. Occasionally, object language variables (\( vv, vv_1, \ldots \)) will also be used as metalinguistic variables. The context should always make clear how variables are being used.
D. A distinguished binary predicate $≺$ representing ‘is one of’ or ‘is among’, hence taking a singular term and a plural variable as arguments.

The recursive clauses defining a well-formed formula are the obvious ones.

A central semantic phenomenon concerning plurals is the distinction between distributive and collective plural predicates. Let $\Phi$ be a plural predicate and let $\varphi$ be its corresponding singular form (if it exists). Then $\Phi$ is said to be *distributive* if

\[
(17) \text{Analytically, for any things } xx, \Phi(xx) \text{ if and only if, for any } y \text{ that is one of } xx, \varphi(y).
\]

A plural predicate is said to be *collective* if it is not distributive.

In the regimenting language just introduced, we can obtain the force of distributive predication without employing distributive predicates. Consider this sentence:

(18) Socrates and Plato are philosophers.

The use of the distributive ‘are philosophers’ can be paraphrased away via the singular ‘is a philosopher’ as shown in (19).

(19) Everything that is one of Socrates and Plato is a philosopher.

For simplicity, we can then expunge distributive predicates from the regimenting language.

The existence of a good method of paraphrase also makes the introduction of a symbol for phrasal conjunction strictly unnecessary. This is shown by the following pair of sentences.

(20) Russell and Whitehead cooperate.

(21) There are some things such that Russell is one of them, Whitehead is one of them, no other thing is one of them, and they cooperate.

Finally, some additional concepts do not require a separate treatment but can be introduced as abbreviations. The plural ‘are among’, symbolized by $\preceq$, can be defined by:

(22) $\forall v_1 \preceq v_2 \leftrightarrow \forall v (v \preceq v_1 \to v \preceq v_2)$.

Plural identity, symbolized by $\approx$, is also definable:

(23) $v_1 \approx v_2 \leftrightarrow v_1 \preceq v_2 \land v_2 \preceq v_1$. 
This completes the presentation of the regimenting language. A language of this sort sides with Regimentation Pluralism but, as I pointed out above, Regimentation Pluralism does not mandate any particular semantic account of plurals and is compatible with Semantic Singularism. Now that we have settled on a regimenting language, we can enter the semantic debate, explore the options, and decide whether any construal of Semantic Singularism is tenable.

4 Semantic approaches to plurals

Semantic singularists and semantic pluralists can agree on a broad characterization of the central notion of truth relative to an interpretation. If they admit properties in their ontology — and this is the case in which we are primarily interested — they will hold that an atomic predication is true in a given interpretation if and only if, relative to that interpretation, the property denoted by the predicate is instantiated by what is denoted by the subject term. As this makes clear, different semantic accounts arise from different ways of assigning denotations to terms and predicates.

Moreover, if properties are admitted, semantic singularists and semantic pluralists will also agree on the specific characterization of truth relative to an interpretation for singular predication, as they will agree that a singular term denotes an object and a singular predicate denotes a singular property. Their disagreement lies in how to specify the denotations of plural terms, which constrains what kind of properties can serve as the denotations of plural predicates. As we will see in a moment, there is also internal disagreement among semantic singularists and among semantic pluralists. Let us start by introducing the main views.

Competing semantics of plurals may be classified along two dimensions. The first dimension concerns how many entities are taken to be denoted by a plural term. In particular, does a plural term denote one or many entities? The second dimension of classification concerns what sort of entities are are taken to be denoted by a plural term. In particular, does a plural term denote an object or a property? These

16Two features of this language should be noted. Since the existence in natural language of genuine plural proper names has been disputed, plural constants have been left out. Definite description operators have also been left out, but for a different reason. Providing an adequate semantics for plural definite descriptions presents distinctive difficulties and goes beyond the scope of this article. In one case (‘the authors of Principia Mathematica’), it will be convenient to assume that a Russellian analysis of the plural description is acceptable. Given that complex plural terms formed by conjoining singular constants are paraphrased away, the only plural terms contained in the language are the plural variables. This parsimony, however, is not pointless. It will expedite the formulation of the semantics in the Appendix. Nothing important for our purposes hinges on these features of the regimenting language.

17For simplicity, I am focusing here on monadic predication. The general case is addressed in section 5 and in the Appendix.
possibilities give rise to four views: object singularism, property singularism, object pluralism, and property pluralism.

<table>
<thead>
<tr>
<th></th>
<th>How many entities are denoted by a plural term?</th>
<th>What sort of entities are denoted by a plural term?</th>
</tr>
</thead>
<tbody>
<tr>
<td>object singularism</td>
<td>one</td>
<td>object</td>
</tr>
<tr>
<td>property singularism</td>
<td>one</td>
<td>property</td>
</tr>
<tr>
<td>object pluralism</td>
<td>many</td>
<td>objects</td>
</tr>
<tr>
<td>property pluralism</td>
<td>many</td>
<td>properties</td>
</tr>
</tbody>
</table>

The last view appears to be the least attractive, since it combines controversial aspects of the other views without offering any advantage over them. So we may take object pluralism to be the only pluralist option and use ‘pluralism’ to mean ‘object pluralism’.

Once the semantic values of plural terms have been determined, one must decide what semantic values should be assigned to plural predicates. There are obvious choices in the singularist camp. On the one hand, an object singularist subscribing to the singular conception will assign singular properties to plural predicates. On the other hand, a property singularist, who takes plural terms to denote properties, will take plural predicates to denote higher-order properties.

In the pluralist camp, three main views have been proposed. According to the first, which may be called untyped pluralism, plural predicates denote plural properties construed as objects, which means that no type distinction between objects and properties is postulated (McKay 2006). An alternative view adopted by Yi (1999, 2002, 2005, 2006) and Oliver and Smiley (2004, 2005, 2006) also takes plural predicates to denote plural properties, but it construes properties as typed. That is, properties are now thought of as entities of a different type than objects. By contrast with the first, we will call this view typed pluralism. Lastly, plural predicates have been taken to denote ‘superpluralities’, i.e., many pluralities at once. This approach, which may be called superpluralism, has been presented by Rayo (2006).

<table>
<thead>
<tr>
<th></th>
<th>What do plural predicates denote?</th>
</tr>
</thead>
<tbody>
<tr>
<td>untyped pluralism</td>
<td>untyped properties</td>
</tr>
<tr>
<td>typed pluralism</td>
<td>typed properties</td>
</tr>
<tr>
<td>superpluralism</td>
<td>‘superpluralities’</td>
</tr>
</tbody>
</table>

Since properties play no role in superpluralism, this view falls outside the main focus of this article, which is the dispute between defenders of the singular conception and advocates of the plural conception. Hence superpluralism and the issues surrounding it (e.g., intelligibility and ontological innocence) will be put aside here. In the remainder of this section, I will discuss some arguments that will help us narrow down
our pool of candidates to two views: property singularism and typed pluralism. In the next two sections, I will compare these views and argue that property singularism provides a satisfactory answer to the semantic challenge.

On the most natural construal of object singularism, plural terms denote non-empty sets and ‘being one of’ is interpreted as set-theoretic membership. The requirement that the sets denoted be non-empty is needed to render ‘there are some things such that nothing is among them’ logically false.\(^\text{18}\) Now, in any interpretation of the language, the term ‘Russell and Whitehead’ denotes the set whose members are the denotation of ‘Russell’ and the denotation of ‘Whitehead’ in that interpretation. Notice that object singularism avoids the objections to REGIMENTATION SINGULARISM discussed in section 2. Since there are interpretations in which ‘Russell and Whitehead cooperate’ is true but ‘there is a set or there is a mereological sum’ is false, the former sentence does not entail the latter.\(^\text{19}\) The existence of a set in the metalanguage is consistent with the falsity of the object language sentence ‘there is a set’. Therefore, the truth of a plural sentence has no implausible consequence concerning sets or mereological sums. The object singularist can sidestep the other objections to REGIMENTATION SINGULARISM by analogous considerations.

However, domains of quantifications are now forced to be set-sized. This sentence follows logically from plural comprehension applied to the formula \(v = v\):

\[
(24) \quad \exists vv \forall v v < vv. \quad \text{\(^\text{20}\)}
\]

If plural terms denote non-empty sets of objects in the range of the first-order quantifiers, plural quantifiers must be taken to range over such sets. Thus, (24) amounts to the requirement that the things in the domain of the first-order quantifiers form a set. So, on the basis of standard set-theoretic assumptions, the first-order domain of quantification cannot be absolutely unrestricted. In particular, it cannot contain every set.

In section 2, it was observed that (3) expresses a truth but, on the set-theoretic version of REGIMENTATION SINGULARISM, it is incompatible with very modest set-theoretic principles.

\(^{18}\)By ruling out singletons, one could accommodate the further condition that ‘there are some thing such that only one thing is among them’ be logically false too. However, this additional condition has been thought to be problematic and will not be assumed here.

\(^{19}\)Consider, for instance, an interpretation in which the extensions of ‘is a set’ and ‘is a sum’ are empty, ‘Russell’ denotes \(r\), ‘Whitehead’ denotes \(w\), and ‘cooperate’ denotes a singular property \(C\) that is instantiated by the set \(\{r, w\}\). Because \(\{r, w\}\) instantiates \(C\), ‘Russell and Whitehead cooperate’ is true in this interpretation, whereas ‘there is a set or there is a mereological sum’ is false.

\(^{20}\)The regimented version of plural comprehension is the axiom schema:

\[(\text{PC}) \quad \exists v \phi(v) \rightarrow \exists vv \forall v (v < vv \leftrightarrow \phi(v)).\]
(3) There are some sets such that any set is one of them if and only if it is not self-membered.

Owing to the distinction between singular and plural terms in the regimenting language, that is no longer the case. If strong enough set-theoretic assumptions are made by the object singularist in the metalanguage to guarantee the existence of a model of the axioms of ZFC, then (3) has a model in conjunction with those axioms. But the domain of the model will still be set-sized.

There is something *prima facie* unsatisfactory with this situation. The sentence is intended to talk about every set there is, not just those contained in some appropriately large set. Object singularism requires that the things in the range of the first order quantifiers form a set and, therefore, is not able to capture models where the domain encompasses every set. The object singularist could resort to classes to provide a domain for such models. But, then, the same problem would arise for another sentence.

(25) There are some classes such that any class is one of them if and only if it is not self-membered.

An appeal to super–classes or some other type of higher-level collection would only delay the problem. Here is how David Lewis puts it.

```
Whatever class-like things there may be altogether, holding none in reserve, it seems we can truly say that there are those of them that are non-self-members. Maybe the singularist replies that some mystical censor stops us from quantifying over absolutely everything without restriction. 21
```

The argument requires that one accept the possibility of absolutely unrestricted quantification, or at least the possibility of quantification over all collections. How could the object singularist do justice to this possibility?

She might retort that the problem has little to do with plurals. It already arises for singular languages within the standard model-theoretic framework. Since domains are sets in that framework, we can never interpret a sentence like (26) as talking about every set there is.

(26) Every set is not self-membered.

---

21 To which he adds: “Lo, he violates his own stricture in the very act of proclaiming it!” Lewis (1991), p. 68.
The mere inability to capture absolutely unrestricted quantification — the object singularist concludes — should not cast doubt on object singularism as a semantic view about plurals.

The burden is now on the opponent of object singularism to show that there is a distinctive difficulty with the inability to achieve absolutely unrestricted quantification in the plural case. A promising way to do that is to reflect on a logical difference between standard semantics for a first-order language and the object singularist semantics for plurals.

A famous argument by Kreisel suggests that, for a first-order language, the semantic notion of consequence and the notion of provability coincide extensionally with the intuitive notion of consequence. That is, despite the fact that the semantics only appeals to models with set-sized domains, the resulting relation of consequence matches the intuitive relation of logical consequence. This, it might be argued, should put to rest worries about absolutely unrestricted quantification in the first-order case. At least for the purposes of characterizing extensionally the notion of logical consequence for a singular language, set-sized domains are enough.

However, it can be proved that the relation of consequence sanctioned by object singularism is not compact. By a standard argument, it follows that there is no effective proof system that is sound and complete for the object singularist semantics. Therefore, Kreisel’s argument is not immediately available to the object singularist. Worries about absolutely unrestricted quantification cannot be easily put to rest in the plural case. Notice that this argument does not threaten property singularism. On this view, (24) expresses the requirement that there is a property instantiated by all the objects in the range of the first-order quantifiers.

(24) $\exists vv \forall v v < vv$.

Nothing prevents the range from being all-inclusive. Any universal property, say that

---

22See Kreisel (1967). This familiar argument, known as the ‘squeezing argument’, turns essentially on the fact that there is a sound and complete proof system for first-order logic equipped with the usual set-theoretic semantics.

23There is an infinite set of sentences $\Gamma$ and a sentence $\sigma$ such that $\Gamma \models \sigma$ but for no finite $\Gamma_0 \subset \Gamma$, $\Gamma_0 \vdash \sigma$. Following Yi (2006), p. 262, here is a proof sketch. For $c_0, c_1, c_2, \ldots$ infinitely many singular constants and $A$ a binary predicate, let $\Gamma$ be the set of sentences $\{A(c_n, c_{n+1}) : n \in \omega\}$. Then

$$\Gamma \models \exists vv \forall v_0 (v_0 < vv \rightarrow \exists v_1 (v_1 < vv \land A(v_0, v_1)))$$

However, there is no finite subset $\Gamma_0$ of $\Gamma$ such that

$$\Gamma_0 \models \exists vv \forall v_0 (v_0 < vv \rightarrow \exists v_1 (v_1 < vv \land A(v_0, v_1)))$$

24Proof sketch. Suppose that $S$ is such a system. Let $\vdash_S$ denote its provability relation. If $\Gamma \models \sigma$ and $S$ is complete, $\Gamma \vdash_S \sigma$. Since $S$ is effective, there is a finite $\Gamma_0 \subset \Gamma$ such that $\Gamma_0 \vdash_S \sigma$. But $S$ is sound, hence $\Gamma_0 \models \sigma$. Therefore, the consequence relation is compact. Contradiction.
of being self-identical, witnesses the plural existential quantifier when the first-order quantifiers range over absolutely everything. Those who wish to admit the possibility of absolutely unrestricted quantification have an argument against object singularism from which property singularism is immune.\(^\text{25}\)

Let us now turn to pluralism. The pluralist who subscribes to the plural conception takes plural terms to denote many things at once and takes plural predicates to denote plural properties. Two versions of this view were identified above. The difference concerns whether a type distinction between objects and properties is postulated.

There are at least two reasons to construe properties as typed. A Russell-style argument put forth by Williamson (2003) shows that, if one accepts an intuitive requirement on what semantic interpretations there are and also admits the possibility of absolutely unrestricted quantification, one cannot construe semantic interpretations as objects. Williamson’s argument can be extended to show that, as long as properties are conceived as objects, one cannot construe semantic interpretations as pluralities either. But this is the natural construal of the notion of an interpretation for the untyped pluralist.\(^\text{26}\) Since allowing for the possibility of quantification over absolutely everything has an important role in the rejection of object singularism, this difficulty for untyped pluralism is especially pressing.

There is an additional problem. Untyped pluralism runs against a plural version of Cantor’s Theorem. Speaking informally, if there is more than one object, there are more pluralities than objects. Thus, if properties are objects, there are more pluralities than properties. It follows that collective plural predicates cannot receive some of their intuitive interpretations. Spelling out these issues would demand more discussion than is possible here and has to be left for another occasion.\(^\text{27}\) Importantly, typed pluralism escapes both problems and, thus, appears to be the best pluralist option. Parallel arguments show that even the property singularist must take properties to be typed.

To sum up, we have found reasons to reject two of the four competing semantic

\(^{25}\)There is controversy over whether absolutely unrestricted quantification can be achieved. See the Introduction to Rayo and Uzquiano (2006) for an overview of the main issues. A number of authors have argued against the possibility of absolutely unrestricted quantification, e.g., Glanzberg (2004, 2006), Hellman (2006), Parsons (2006), and Weir (2006). Since rejecting absolutely unrestricted quantification would remove the main difficulty with object singularism and weaken the case for pluralism and for the plural conception, I will assume that absolutely unrestricted quantification can be achieved. Perhaps, invoking suitable reflection principles can help the object singularist respond to the argument. However, that is a cost not incurred by the property singularist.

\(^{26}\)See McKay (2006), pp. 147-154, for a discussion of Williamson’s argument from the perspective of untyped pluralism. This view construes semantic interpretations as pluralities in the sense that they are given by *some things*, specifically ordered pairs, coding information about the domain and about the denotations of terms and predicates.

\(^{27}\)I offer a detailed formulation of these criticisms in my “Untyped Pluralism” (work in progress).
accounts of plurals under consideration, i.e., object singularism, property singularism, untyped pluralism, and typed pluralism. If we want to do justice to absolutely unrestricted quantification, there is pressure to reject object singularism. Moreover, paradox and cardinality considerations undermine untyped pluralism. This leaves us with two contenders, one in the singularism camp and one in the pluralist camp: property singularism and typed pluralism. In the next section, I will compare these two accounts. Then, I will defend property singularism from potential objections.

5 Typed pluralism and property singularism

Although it encompasses infinitely many types of entities, the hierarchy underpinning the singularist semantics has a simple structure: there are objects at the bottom (entities of type 0), then there are singular properties of objects (type 1), singular properties of singular properties of objects (type 2), and so on. Of course, one must countenance relational properties as well as monadic ones. This hierarchy corresponds to the hierarchy of standard higher-order logic generated from first-order logic. To highlight the connection with higher-order logic, properties of type \( n \) will be called \( n \)-order properties. A superscript will indicate the type of an entity in the singular hierarchy: objects will be indicated by the superscript 0 (e.g., \( x^0, y^0, z^0 \)), first-order properties by the superscript 1 (e.g., \( x^1, y^1, z^1 \)), and second-order properties by the superscript 2 (e.g., \( x^2, y^2, z^2 \)).

The hierarchy underpinning the typed pluralist semantics is bifurcated. It includes the singular hierarchy just outlined plus a parallel hierarchy that includes plural properties (indicated by Greek letters \( \alpha, \beta, \ldots \)), properties of plural properties, and so on. Luckily, the semantic views under consideration require entities of a few low types, and so we can avoid most issues concerning the exact construction of the hierarchies. However, some assumptions should be mentioned.

First, we may assume that genuinely plural properties are found only at the initial stage of the plural hierarchy. For our purposes, there is no need to introduce higher-order plural properties, that is, higher-order properties instantiated by many properties collectively. Second, it is plausible to assume that a property can only be instantiated by (or relate) entities of lower types, but we do not require that all of its arguments be of the type immediately below. In other words, we take the hierarchies to be cumulative. Moreover, in conformity with the Fregean tradition, it is customary to assume that an argument place of a property can be occupied by enti-

\[28\] For terminological economy, I use ‘property’ to include relations (‘relational properties’) as well as monadic properties.

\[29\] The formal treatment in the Appendix makes clear what types of entities are involved in the semantics, both in the specification of the semantic values and in the characterization of the basic semantic relations.
ties of at most one type. We call this assumption *exclusivity*. A property that violates exclusivity will be called *multifaceted*. Incidentally, it is worth pointing out that exclusivity introduces some complications in the semantics. Suppose that we want to define a general notion of interpretation as a relation between linguistic objects (type 0) and their semantic values. The typed pluralist, for example, would want to interpret a plural term $vv$ as denoting some things $xx$ in the domain, so she would have to characterize interpretations as *plural* relational properties. However, she would also want to interpret a singular term $t$ as denoting an object in the domain. This means that interpretations would also have to be regarded as *singular* relational properties. If exclusivity is dropped, a multifaceted property could do double duty and help avoid the multiplication of concepts, namely, the need to introduce a distinct notion of interpretation for each category of non-logical vocabulary. A single multifaceted property would then be able to relate two objects (a singular term and its denotation) as well as an object and a plurality (a plural variable and the things it denotes). A similar complication is forced by exclusivity in the context of property singularism. An argument for multifaceted properties based on natural language considerations will be presented in section 6.\(^3\)

As noted above, typed pluralism and property singualrism agree on the semantics for the singular fragment of the language, taking a singular term to denote an object and taking a singular predicate to denote a singular property. For the pluralist, plural terms denote pluralities and plural predicates denote plural properties, whereas for the property singularist plural terms denote singular first-order properties and plural predicates denote singular second-order properties. However, two restrictions are needed in the property singularist semantics. The semantic values of plural terms should be confined to *non-empty* properties and those of plural predicates should be confined to second-order *extensional* properties. A second-order property is extensional just in case, if it is instantiated by a first-order property $x^1$, it is instantiated by any first-order property coextensive with $x^1$.\(^4\)

The reason for requiring that plural terms denote non-empty properties was mentioned when discussing the analogous requirement for object singularism that plural terms denote non-empty sets. (The semantics must yield that ‘some things are such that nothing is among them’ is logically false.) As for the extensionality requirement, the following sentence would be consistent without it.

\[(27) \quad \exists vv_1 \left( C(vv_1) \land \exists vv_2 \left( \neg C(vv_2) \land vv_1 \approx vv_2 \right) \right).\]

But it should not be, since it regiments ‘there are some people who cooperate and

\(^{30}\)In order to provide a more neutral formulation of the semantics, the formal treatment in the Appendix will not employ multifaceted properties.

\(^{31}\)More generally, an $n$-ary second-order property $x^2$ is said to be extensional just in case, for every $x^1_1, \ldots, x^1_{i}, \ldots, x^1_m$ (1 ≤ $i$ ≤ $m$) and $x^0_{1}, \ldots, x^0_{n-m}$, if $x^1_i$ is coextensive with $y^1$ and $x^2(x^1_1, \ldots, x^1_{i}, \ldots, x^1_m, x^0_1, \ldots, x^0_{n-m})$, then $x^2(x^1_1, \ldots, y^1, \ldots, x^1_m, x^0_1, \ldots, x^0_{n-m})$.\)
there are some people who do not cooperate but who are the same as those who cooperate’. To avoid the extensionality requirement, one might interpret \( \approx \) as a primitive symbol standing for property identity. This would render (27) logically false no matter what second-order property is denoted by \( C \). However, the following sentence should have the same logical status of (27).

\[
(28) \exists vv_1 (C(vv_1) \land \exists vv_2 (\neg C(vv_2) \land \forall v (v \prec vv_1 \iff v \prec vv_2))).
\]

But, without the extensionality requirement, (28) would be consistent. So appealing to plural identity as primitive does not help avoid the extensionality requirement.

As further illustration of the two semantics, let us examine how they specify the conditions of truth relative to an interpretation in the case of a basic plural predication, such as ‘Plato and Aristotle disagree’. The regimentation of this sentence is as follows:

\[
(29) \exists vv (\forall v (v \prec vv \iff (v = p \lor v = a)) \land D(vv)).
\]

Let us use subscripts to indicate the relativization of a notion to a given interpretation. On the typed pluralist approach, ‘Plato and Aristotle disagree’ is true in a given interpretation \( I \) if and only if there are some things \( xx \) in the domain \( I \) such that anything is one of them just in case it is the denotation \( I \) of ‘Plato’ or the denotation \( I \) of ‘Aristotle’, and those things instantiate the plural property \( \alpha \) denoted \( I \) by ‘disagree’, i.e., \( \alpha(xx) \).

On the property singularist approach, ‘Plato and Aristotle disagree’ is true in a given interpretation \( J \) if and only if there is a non-empty property \( x^1 \) such that it holds of nothing but the denotation \( J \) of ‘Plato’ and the denotation \( J \) of ‘Aristotle’, and it instantiates the second-order extensional property \( x^2 \) denoted \( J \) by ‘disagree’, i.e., \( x^2(x^1) \).

Typed pluralism and property singularism characterize the notion of truth relative differently, but do they sanction different logics for plurals? The answer is negative. As proved in the Appendix, property singularism yields the same relation of logical consequence for the regimenting language as does typed pluralism. Property singularism relies on resources that are deemed legitimate by the proponent of the plural conception, who admits the existence of both singular and plural properties. The result then shows that plural resources are semantically dispensable, even by the lights of the proponent of the plural conception. We should conclude that the semantic challenge to the singular conception is met unless property singularism is found to be problematic in some respect.
6 Is there a problem with property singularism?

The idea of using properties in connection with plural terms is not new. It may already be found in Russell and it has been suggested repeatedly since then. However, what has usually been proposed — let us call it the traditional approach — differs from property singularism in a critical way. On the one hand, the traditional approach pursues regimentation singularism, employing the language of higher-order logic for regimentation. On the other hand, property singularism is a form of semantic singularism. Properties are invoked only in the interpretations of the language and do not surface at the level of regimentation. In this section, I will discuss three potential problems that might be thought to undermine property singularism, the extensionality problem, the coordination problem, and the homophonicity problem. I will argue that none of them poses a serious threat.

The extensionality problem concerns the legitimacy of letting extensional properties serve as semantic values of plural predicates. Yi (1999, 2005) has raised a version of this problem for the traditional approach. Let us examine his argument to see whether it applies to property singularism. Consider this inference:

Russell and Whitehead cooperate
The authors of *Principia Mathematica*

(30) Russell and Whitehead cooperate
The authors of *Principia Mathematica* cooperate

On the traditional approach, a natural regimentation of (30) is given by (31).

\[ \exists X (\forall x (Xx \leftrightarrow (x = r \lor x = w)) \land C(X)) \]

(31) \[ \exists X (\forall Y (A(Y, p) \leftrightarrow Y = X) \land \forall x (Xx \leftrightarrow (x = r \lor x = w))) \]

\[ \exists X (\forall Y (A(Y, p) \leftrightarrow Y = X) \land C(X)) \]

But (31) is invalid, and it would need to be supplemented with the assumption that cooperating is an extensional second-order property:

(32) \[ \forall X \forall Y ((C(X) \land \forall x (Xx \leftrightarrow Yx)) \rightarrow C(Y)). \]

Since the original inference is logically valid as it stands, namely, without additional premises, (32) must be taken to be a logical truth. As analogous inferences would show, the same considerations extend to other collective predicates in the language. Yi writes:

---


It is one thing to hold the extensional conception, quite another to hold, more implausibly, that the truth of the conception rests on logic alone. [...] One cannot meet the objections [...] under the assumption that the property indicated by “COOPERATE” is one that Russell calls extensional (that is, a second-order property instantiated by any first-order property coextensive with one that instantiates it). This does not help unless the assumption holds by logic [...].

The crucial observation is that this difficulty does not arise for property singularity. This view validates (30) without the need of any extra assumption. The

34Yi (2005), p. 475. A similar remark is in Yi (1999), p. 173. The problem also applies to this formalization of the inference:

\[
\begin{align*}
&\exists X (\forall x (Xx \leftrightarrow (x = r \lor x = w)) \land C(X)) \\
&(\exists X A(X,p) \land \forall x (Yx \leftrightarrow Yx) \land x \leftrightarrow (x = r \lor x = w)) \\
&\exists X (A(X,p) \land \forall x Yx \lor Yx) \land C(X))
\end{align*}
\]

where \(=\) stands for property coextensionality. Without the assumption that cooperating is extensional, the inference remains invalid. Notice, though, that there is an alternative formalization that renders the inference valid without the assumption.

\[
\begin{align*}
&\exists X (\forall x (Xx \leftrightarrow (x = r \lor x = w)) \land C(X)) \\
&(\exists X (\forall x Yx \leftrightarrow Yx) \land x \leftrightarrow (x = r \lor x = w)) \\
&\exists X (\forall Yx Yx \lor Yx) \land C(X))
\end{align*}
\]

One might complain, perhaps, that the assumption of extensionality is now built in with respect to the predicate \(A\). One might also object that ‘Russell and Whitehead’ should not be analyzed but symbolized as the name of a property.

35Let us verify this claim. In the language presented in §2, the inference may be regimented as (33).

\[
\begin{align*}
&\exists v \forall x (v \leftrightarrow x) \land C(v) \\
&\exists v (v_1 (\forall x v_1 (v_2 (v_1 \land v_2) \land v_1 (v_2 (v_1 \land v_2)))) \\
&\exists v (v_1 (v_2 (v_1 \land v_2)) \land v_1 (v_2 (v_1 \land v_2))))
\end{align*}
\]

Let \(\mathcal{I}\) be any interpretation of the language according to property singularity, and let \([p]_I\), \([w]_I\), \([p]_I\) be, respectively, the denotations of ‘Russell’, ‘Whitehead’, and ‘Principia Mathematica’ according to \(\mathcal{I}\). Also, let \([[C]]_I\) and \([[A]]_I\) be, respectively, the singular second-order extensional property denoted by ‘cooperate’ and the extensional (relational) property denoted by ‘are authors of’ in \(\mathcal{I}\). Suppose that the premises are true in \(\mathcal{I}\). Now, the first premise is true in \(\mathcal{I}\) just in case there is a singular first-order property \(x^1\) such that

1a) for every \(x^0\) in the domain, \(x^1(x^0)\) if and only if \(x^0 = [p]_I\) or \(x^0 = [w]_I\);

1b) \([[C]]_I(x^1)\).

The second premise is true in \(\mathcal{I}\) just in case there is a property \(y^1\) satisfying the following conditions:

2a) \([[A]]_I(y^1), [p]_I);\n
2b) for every \(z^1\), \([[A]]_I(z^1, [p]_I)\) only if \(z^1\) is coextensive with \(y^1);\n
2c) for every \(z^0\) in the domain, \(y^1(z^0)\) if and only if \(z^0 = [p]_I\) or \(z^0 = [p]_I\).
requirement that plural predicates be interpreted by second-order extensional properties salvages the validity of (30), but is it acceptable? The property singularist does not have to embrace an extensional conception of properties, let alone embrace it as logically true. On semantic grounds, she just has to confine the semantic values of plural predicates to extensional properties. As it happens, there is no shortage of extensional properties. It follows from the principle of second-order comprehension that there is an extensionalization of any second-order property $z^2$:

$\exists x^2 \forall y^1 (x^2(y^1) \leftrightarrow \exists z^1 (z^2(z^1) \land \forall x^0 (y^1(x^0) \leftrightarrow z^1(x^0))))$.

So the problem is not whether extensional properties are available. Rather, it whether one may impose a restriction on the semantic values of plural predicates which rules out non-extensional properties. As far as I can see, there is nothing that advises against the restriction. It is no more problematic than requiring that plural terms be interpreted as denoting non-empty properties or sets.

According to the second problem — the coordination problem — property singularism generates a mismatch of types among predicates. Consider these two sentences:

(35) (a) Russell and Whitehead wrote a book.

(b) Wittgenstein wrote a book.

From them one may infer (36).

(36) Russell and Whitehead wrote a book and Wittgenstein did too.

The possibility of coordination displayed in (36) might be taken to support the claim that the predicate ‘wrote a book’ occurs univocally in (35a) and (35b) and, thus, it should be assigned the same semantic value. However, for the property singularist the predicate ‘wrote a book’ denotes a second-order property in one case and a first-order property in the other. Notice that the same difficulty is faced by typed pluralism. The typed pluralist assigns a plural property to the predicate in (35a) and a singular property to the predicate in (35b). So this is a prima facie problem for both views.

A number of other tests could be used to argue in favor of the univocity claim. For example, one can quantify over both predicates.

So there is a first-order property, namely $y^1$, such that

(i) $y^2(y^1, [p]_I)$, by (2a);

(ii) for every $z^1$, $[A]_I(z^1, [p]_I)$ only if $z^1$ is coextensive with $y^1$, by (2b);

(iii) $[C]_I(y^1)$, by (1a), (2c), (1b), and the extensionality of $[C]_I$.

Therefore, the conclusion is true in $I$. Since $I$ is arbitrary, the inference is valid.

$^{36}$See Yi (1999), p. 185, for a version of this problem targeted at the traditional conception.
Wittgenstein did something that Russell and Whitehead did.

Moreover, the predicate can be used with disjunctive noun phrases with a plural and singular component, such as ‘two famous British logicians or Wittgenstein’:

Two famous British logicians or Wittgenstein wrote a book.

Furthermore, both ‘Russell and Whitehead (did)’ and ‘Wittgenstein (did)’ would be appropriate answers to the question: ‘Who wrote a book?’.

Fortunately, we do not need to take a stand as to whether these tests provide conclusive evidence for the univocity claim. Even if there is reason to accept the univocity claim, both the typed pluralist and the property singularist could easily accommodate it. They could both drop exclusivity and allow for multifaceted properties. The typed pluralist would have to allow for multifaceted properties that can take both objects and pluralities as arguments, whereas the property singularist would have to allow for multifaceted second-order properties that can take both objects and first-order properties as arguments. An alternative, less attractive response would be to raise the type of singular terms to match those of plural ones. Here the typed pluralist would have to take ‘Wittgenstein’ to denote a plurality composed of one object, whereas the property singularist would have to take it to denote a property that has only one object in its extension. In either case, both semantics have the resources to accommodate coordination phenomena.

Finally, the homophonicity problem stems from the thought that the semantic values of an expression should be homophonic to the expression itself. Typed pluralism would seem to have an advantage over property singularism, since the semantic value of a plural term is given by a plural term in the metalanguage. It is important to distinguish this thought from the objection that, by interpreting plural terms as properties, the property singularist will license incorrect inferences, such as ‘Russell and Whitehead wrote Princípios de uma Teoria das Matemáticas, therefore a property wrote Princípios de uma Teoria das Matemáticas’. The result proved in the Appendix implies that the inference is validated by property singularism (if and) only if it is validated by pluralism. But it would be easy to verify that the inference is simply invalid according to property singularism.

If we are to do justice to the possibility of absolutely unrestricted quantification, the semantics of the object language must be given in a metalanguage that can quantify over entities of a higher type than those over which the object language can quantify. In particular, the entities serving as semantic values of plural predicates cannot

---

37 Thanks to Ben Caplan and Alex Oliver here. For discussion, see Rumfitt (2005) and Oliver (unpublished).

38 See Linnebo and Rayo (forthcoming), Appendix B, for a presentation and discussion of the result. We encountered an instance of it in the rejection of untyped pluralism.
be in the range of the object language quantifiers, singular or plural. This undercuts the general project of giving a homophonic semantics for the object language. The choice of semantic values can be homophonic at best with respect to terms. When predicates are nominalized, e.g., one moves from ‘Plato and Aristotle disagree’ to ‘Plato and Aristotle have the property of disagreeing’, the description ‘the property of disagreeing’ cannot capture the semantic value of the predicate ‘disagree’.

If a fully homophonic interpretation of the non-logical vocabulary of the object language cannot be obtained, it is even more problematic to insist on a requirement that rules out property singularism at the outset. The proponent of the singular conception of reality aims to undermine the semantic argument in favor of the plural conception by showing how, without employing plural resources, an adequate semantics of plurals can be developed. Requiring the use of plural resources is not dialectically legitimate in this context.

7 Conclusion

Semantic considerations about plurals have led some to embrace the plural conception of reality. Plurals do indeed pose a semantic challenge for those who subscribe to the singular conception of reality. Familiar responses to the challenge rely on REGIMENTATION SINGULARISM, which is quite problematic. However, REGIMENTATION SINGULARISM is not needed to provide a satisfactory semantics for plurals. Adopting a regimenting language that includes both singular and plural expressions is compatible with the view that interpretations for such a language can be specified adequately within the boundaries of the singular conception.

Four competing semantic accounts of plurals were examined: object singularism, property singularism, untyped pluralism, and typed pluralism. Especially if one admits the possibility of quantifying over absolutely everything, property singularism and typed pluralism appear to be the most promising views. Typed pluralism invokes plural properties, whereas property singularism does not. Nonetheless, as proved in the Appendix, the two accounts deliver the same relation of logical consequence for the common regimenting language. Thus, no deep semantic advantage is gained by resorting to plural properties. Since property singularism employs resources that are deemed legitimate by those who embrace the plural conception, the adequacy of property singularism shows that, even by the lights of a proponent of the plural conception, plural properties are semantically dispensable. The singular conception of reality is rich enough to provide a satisfactory semantics for plurals. Semantic considerations do not give us reasons to abandon it.
Appendix

After introducing some preliminary notions (Appendix A), I provide a more rigorous presentation of typed pluralism (Appendix B1) and property singularism (Appendix B2). Under minimal assumptions about the relationship between pluralities and properties, I then prove that the two approaches yield the same relation of logical consequence with respect to the common regimenting language (Appendix C).

Appendix A: Preliminary notions

The notation for entities of various types was introduced in section 5. Some abbreviations will be convenient:

\[
\begin{align*}
x^1 & \subseteq y^1 \overset{\text{def}}{=} \forall z^0 (x^1(z^0) \rightarrow y^1(z^0)), \\
x x & \preceq x^1 \overset{\text{def}}{=} \forall z^0 (z^0 \prec xx \rightarrow x^1(z^0)), \\
x x & \equiv x^1 \overset{\text{def}}{=} \forall z^0 (z^0 \prec xx \leftrightarrow x^1(z^0)), \\
\alpha \times^n[m] x^2 & \overset{\text{def}}{=} \forall x_1, \ldots, x_m \forall x^1_1, \ldots, x^1_m \forall x^0_1, \ldots, x^0_{n-m} \left((\alpha(x_1, \ldots, x_m, x^0_1, \ldots, x^0_{n-m}) \& xx \equiv x^1_1 \& \ldots \& xx \equiv x^1_m) \rightarrow x^2(x^1_1, \ldots, x^1_m, x^0_1, \ldots, x^0_{n-m})\right), \\
\alpha \succ^n[m] x^2 & \overset{\text{def}}{=} \forall x^1_1, \ldots, x^1_m \forall x_1, \ldots, x_m \forall x^0_1, \ldots, x^0_{n-m} \left((x^2(x^1_1, \ldots, x^1_m, x^0_1, \ldots, x^0_{n-m}) \& xx \equiv x^1_1 \& \ldots \& xx \equiv x^1_m) \rightarrow \alpha(x_1, \ldots, x_m, x^0_1, \ldots, x^0_{n-m})\right), \\
\alpha \bowtie^n[m] x^2 & \overset{\text{def}}{=} \alpha \times^n[m] x^2 \& \alpha \succ^n[m] x^2.
\end{align*}
\]

Let us define the basic semantic notions. First, we have relations of interpretation \(I\) and assignment \(A\) between constants or variables of the regimenting language and entities populating the universe of the metatheory. Since interpretations and assignments are taken to be functional, we may use the square bracket notation \([s]_R\) to indicate the unique entity (or entities) assigned to \(s\) by \(R\). The typed pluralist and the property singularist agree on the semantics of the singular fragment of the language and diverge on the plural fragment. So interpretations of singular constants \((I_0)\) and interpretations of singular predicates \((I_1)\), as well as assignments for singular variables \((A_0)\), will be the same for both accounts. These semantic relations are
characterized by the following conditions on \( I_0, I_1, \) and \( A_0, \) relative to a domain of quantification given by a singular property \( d^1.\)

1. For any singular constant \( c, \) there is \( x^0 \) such that \([c]_{I_0} = x^0 \) and \( d^1(x^0). \)

2. For any singular predicate \( S, \) there is \( x^1 \) such that \([S]_{I_1} = x^1 \) and, for every \( z^0_1, \ldots, z^0_n, \) if \( x^1(z^0_1, \ldots, z^0_n), \) then \( d^1(z^0_1), \ldots, d^1(z^0_n). \)

3. For any singular variable \( v, \) there is \( x^0 \) such that \([v]_{A_0} = x^0 \) and \( d^1(x^0). \) A variant \( A_0[v/y^0] \) of \( A_0 \) with respect to \( v \) is defined in the usual way.

According to typed pluralism, interpretations of plural predicates \((I_2)\) assign a plural property to each plural predicate. Variable assignments for plural variables \((A_1)\) assign some things in the domain to each plural variable. That is, relative to a domain \( d^1, \) we have the following conditions on \( I_2 \) and \( A_1.

4. For any plural predicate \( P^{n[m]}, \) there is \( \alpha \) such that \([P^{n[m]}]_{I_2} = \alpha \) and, for every \( z^1_1, \ldots, z^1_m, z^0_1, \ldots, z^0_{n-m}, \) if \( \alpha(z^1_1, \ldots, z^1_m, z^0_1, \ldots, z^0_{n-m}), \) then \( z^1_1 \sqsubseteq d^1, \ldots, z^0_{n-m} \sqsubseteq d^1 \) and \( d^1(z^0_1), \ldots, d^1(z^0_{n-m}). \)

5. For any plural variable \( vv, \) there are \( xx \) such that \([vv]_{A_1} \approx xx \) and \( xx \sqsubseteq d^1. \) A variant \( A_1[vv/vy] \) of \( A_1 \) with respect to \( vv \) is also defined in the usual.

Next we have property singularist counterparts of these semantic relations, namely, interpretations of plural predicates \((I_2^*)\) assigning second-order extensional properties to each plural predicate, and variable assignments for plural variables \((A_1^*)\) assigning singular properties to each plural variable.

4*. For any plural predicate \( P^{n[m]}, \) there is an extensional \( x^2 \) such that \([P^{n[m]}]_{I_2^*} = x^2 \) and, for every \( z^1_1, \ldots, z^1_m, z^0_1, \ldots, z^0_{n-m} (m \leq n), \) if \( x^2(z^1_1, \ldots, z^1_m, z^0_1, \ldots, z^0_{n-m}), \) then \( z^1_1 \sqsubseteq d^1, \ldots, z^0_{n-m} \sqsubseteq d^1 \) and \( d^1(z^0_1), \ldots, d^1(z^0_{n-m}). \)

5*. For any plural variable \( vv, \) there is a non-empty \( x^1 \) such that \([vv]_{A_1^*} = x^1 \) and \( x^1 \sqsubseteq d^1. \) A variant \( A_1^*[vv/vy] \) of \( A_1^* \) with respect to \( vv \) must satisfy the extra condition that \( y^1 \) be non-empty.

An additional abbreviation will be useful. For any singular term \( t, \)

\[
[t]_{A_0/I_0} = \begin{cases} [t]_{A_0} & \text{if } t \text{ is a variable,} \\
[t]_{I_0} & \text{if } t \text{ is a constant.}
\end{cases}
\]

We are now ready to formulate the two semantics.

\textit{39}For the typed pluralist, domains of quantifications are more naturally given by some \textit{things}. We could accommodate this option but defining domains in terms of singular properties simplifies the exposition. That will not do extreme violence to typed pluralism, since the typed pluralist does not reject singular properties.
Appendix B1: Typed pluralism

The typed pluralist notion of satisfaction, $\text{Sat}^t(\phi, I_0, I_1, I_2, A_1, A_2)$ (rewritten as $I_0, I_1, I_2 \models \phi [A_0, A_1]$), is implicitly defined by the following clauses relative to interpretations and assignments all defined on the same domain $d^1$.

(i) If $\phi$ is $t = s$, $I_0, I_1, I_2 \models \phi [A_0, A_1]$ if and only if $[t]_{A_0/I_0} = [s]_{A_0/I_0}$.

(ii) If $\phi$ is $t < vv$, $I_0, I_1, I_2 \models \phi [A_0, A_1]$ if and only $[t]_{A_0/I_0} < [vv]_{A_1}$.

(iii) If $\phi$ is $S^n(t_1, \ldots, t_n)$, $I_0, I_1, I_2 \models S^n(t_1, \ldots, t_n) [A_0, A_1]$ if and only if

$$[S^n]_{I_1} ([I_1]_{A_0/I_0}, \ldots, [I_n]_{A_0/I_0}).$$

(iii) If $\phi$ is $P^{nm}[v_1, \ldots, v_m, t_1, \ldots, t_{n-m}]$,

$$I_0, I_1, I_2 \models P^{nm}[v_1, \ldots, v_m, t_1, \ldots, t_{n-m}) [A_0, A_1] \text{ if and only if}
\[[P^{nm}]_{I_2} ([v_1]_{A_1}, \ldots, [v_m]_{A_1}, [t_1]_{A_0/I_0}, \ldots, [t_{n-m}]_{A_0/I_0}).$$

(iv) If $\phi$ is $\exists y \psi$, $I_0, I_1, I_2 \models \phi [A_0, A_1]$ if and only if, for some $x^0$ such that $d^1(x^0)$,

$$I_0, I_1, I_2 \models \psi [A_0[v/x^0], A_1].$$

(v) If $\phi$ is $\exists vv \psi$, $I_0, I_1, I_2 \models \phi [A_0, A_1]$ if and only if, for some $xx$ such that $xx \leq d^1$,

$$I_0, I_1, I_2 \models \psi [A_0, A_1[vv/xx]].$$

(vi-xi) The clauses for negation and for the binary connectives are the obvious ones.

The final step is to characterize the notions of logical consequence and logical truth. For any set of sentences $\Gamma$ and any sentence $\sigma$, $\sigma$ is a logical consequence of $\Gamma$ according to typed pluralism ($\Gamma \models_{TP} \sigma$) just in case for any interpretations $I_0, I_1, I_2$ and assignments $A_0, A_1$, if $I_0, I_1, I_2 \models \gamma [A_0, A_1]$ for any member $\gamma$ of $\Gamma$, then $I_0, I_1, I_2 \models \sigma [A_0, A_1]$.

Appendix B2: Property singularism

The property singularist notion of satisfaction, $\text{Sat}^s(\phi, I_0, I_1, I_2, A_1, A_2^s)$ (rewritten as $I_0, I_1, I_2^s \models^* \phi [A_0, A_1^s]$), is also defined implicitly with respect to interpretations and assignments defined on the same domain $d^1$. The satisfaction clauses for the singular fragment of the language overlap with the typed pluralist clauses. The clauses for the plural fragment of the language are as follows.

(ii) If $\phi$ is $t < vv$, $I_0, I_1, I_2^s \models^* \phi [A_0, A_1^s]$ if and only if $[vv]_{A_1^s([I]_{A_0/I_0})}$. 


If $\phi$ is $P^{[m]}(vv_1, \ldots, vv_n, t_1, \ldots, t_{n-m})$,

$$I_0, I_1, I_2^* \models^* \phi [A_0, A_1^*] \text{ if and only if }$$

$$[P^{[m]}]_{I_2}([vv_1]_{A_1^*}, \ldots, [vv_n]_{A_1^*}, [I_1]_{A_0/I_0}, \ldots, [I_{n-m}]_{A_0/I_0}).$$

If $\phi$ is $\exists vv \psi$, $I_0, I_1, I_2^* \models^* \phi [A_0, A_1^*]$ if and only if, for some non-empty $x^1$ such that $x^1 \sqsubseteq d^1$, $I_0, I_1, I_2^* \models^* \psi [A_0, A_1^*[vv/x^1]].$

The property singularist notions of logical consequence ($\Gamma \models_{PS} \sigma$) and logical truth are then characterized in the obvious way by quantifying over interpretations.

**Appendix C: The equivalence**

For any two interpretations $I_2$ and $I_2^*$ and for any two assignments $A_1$ and $A_1^*$ all defined on the same domain, the notation $\Phi(I_2, I_2^*: A_1, A_1^*)$ will abbreviate the following: for any plural predicate of the language $P^{[m]}$, $[P^{[m]}]_{I_2} \times [m] [P^{[m]}]_{I_2^*}$ and, for any plural variable $vv$, $[vv]_{A_1} \equiv [vv]_{A_1^*}.$

Here are the assumptions used to prove the main result below. They sanction a correspondence in extension between pluralities and non-empty singular first-order properties, and between plural properties and singular second-order ones.

1. $\forall xx \exists x^1 \ x x \equiv x^1.$
2. $\forall x^1 (\exists x^0 x^1(x^0) \rightarrow \exists xx \ x x \equiv x^1).$
3. For every $n, m$ such that $n \geq m \geq 1$, $\forall \alpha \exists x^2 \ \alpha \times^{n,m} x^2.$
4. For every $n, m$ such that $n \geq m \geq 1$, $\forall x^2 \ \exists \alpha \ \alpha \times^{n,m} x^2.$

These assumptions are certainly problematic if properties are construed as objects. In that case, a plural version of Cantor’s Theorem would bar the assumptions. But once properties are typed, the obstacle is removed.

**Lemma 1.** *Lemma* For any formula $\phi$, interpretations $I_2$ and $I_2^*$, and assignments $A_1$ and $A_1^*$, if $\Phi(I_2, I_2^*: A_1, A_1^*)$, then for every $I_0$, $I_1$ and $A_0$,

$$I_0, I_1, I_2 \models^* \phi [A_0, A_1] \text{ if and only if } I_0, I_1, I_2^* \models^* \phi [A_0, A_1^*].$$

**Proof.** By induction on the complexity of $\phi$. If $\phi$ is singular, the claim is trivial. If $\phi$ is $t \prec vv$, then

$$I_0, I_1, I_2 \models^* \phi [A_0, A_1] \iff (\because \Phi(I_2, I_2^*: A_1, A_1^*), \ [vv]_{A_1} \equiv [vv]_{A_1^*})$$

$$\iff \lang [I]_{A_0/I_0} \prec [vv]_{A_1} \lang \iff I_0, I_1, I_2^* \models^* \phi [A_0, A_1^*].$$
If $\phi$ is $P^{[m]}(v_1, \ldots, v_m, t_1, \ldots, t_{n-m})$, then

$$I_0, I_1, I_2 \models P^{[m]}(v_1, \ldots, v_m, t_1, \ldots, t_{n-m}) [A_0, A_1] \iff \left[ P^{[m]} \right]_{I_2} (\left[ [v_1] A_1, \ldots, [v_m] A_1, [t_1] A_0 / I_0, \ldots, [t_{n-m}] A_0 / I_0 \right) \iff (\text{since } \Phi(I_2, I_2^*; A_1, A_1^*), \right.$$}

$$\left[ [v_i] A_1 \equiv [v_i] A_1^*, \right.$$}

$$\left[ P^{[m]} \right]_{I_2} \models \left[ P^{[m]} \right]_{I_2} \right) \iff I_0, I_1, I_2^* \models P^{[m]}(v_1, \ldots, v_m, t_1, \ldots, t_{n-m}) [A_0, A_1^*].$$

Finally, if $\phi$ is $\exists v \psi$, then

$$I_0, I_1, I_2 \models \exists v \psi [A_0, A_1] \iff \text{for some } xx \subseteq d^1, I_0, I_1, I_2 \models \exists v \psi [A_0, A_1[vv/xx]] \iff (\text{by (1) and (2)})$$

$$\text{for some non-empty } x^1 \subseteq d^1, I_0, I_1, I_2^* \models \psi [A_0, A_1^*[vv/x^1]] \iff I_0, I_1, I_2^* \models \exists v \psi [A_0, A_1^*].$$

This completes the induction. 

On the basis of (1)–(4), the following is clear.

**Lemma 2.** For any $I_2$ and $A_1$ defined on the same domain, there are $I_2^*$ and $A_1^*$ such that $\Phi(I_2, I_2^*; A_1, A_1^*)$. Conversely, for any $I_2^*$ and $A_1^*$ defined on the same domain, there are $I_2$ and $A_1$ such that $\Phi(I_2, I_2^*; A_1, A_1^*)$.

Now we can prove the result we are after.

**Proposition.** For any sentence $\sigma$ and any set of sentences $\Gamma$,

$$\Gamma \models_{TP} \sigma \text{ if and only if } \Gamma \models_{PS} \sigma.$$

**Proof.** Suppose that $\Gamma \models_{TP} \sigma$. Let $I_0, I_1, I_2, A_0, A_1^*$ be arbitrary interpretations and assignments such that $I_0, I_1, I_2^* \models \gamma [A_0, A_1^*]$ for every member $\gamma$ of $\Gamma$. By Lemma 2, $\Phi(I_2, I_2^*; A_1, A_1^*)$ for some $I_2$ and $A_1$. By Lemma 1, $I_0, I_1, I_2 \models \gamma [A_0, A_1]$ for every member $\gamma$ of $\Gamma$. But $\Gamma \models_{TP} \sigma$, hence $I_0, I_1, I_2 \models \sigma [A_0, A_1]$. By Lemma 1 again, $I_0, I_1, I_2^* \models \sigma [A_0, A_1^*]$. Since $I_0, I_1, I_2, A_0, A_1^*$ are arbitrary, $\Gamma \models_{PS} \sigma$. The other direction is proved similarly.
References

Boolos, G.: 1984, To be is to be a value of a variable (or to be some values of some variables), *Journal of Philosophy* **81**, 430–450.


Nicolas, D.: unpublished, Can mereological sums serve as the semantic values of plurals?

Oliver, A.: unpublished, Against predicative theories of plurality.


